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Pionic degrees of freedom in atomic nuclei and quasielastic knockout of pions by high-energy electrons

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Abstract. Two kinds of virtual pions in the nucleus are discussed: delocalized pions forming a pionic condensate and pions localized in the pionic clouds of individual nucleons in the nucleus. The pionic degrees of freedom in nuclei may be efficiently verified in pion quasielastic knockout experiments $A(e, e'\pi^{\pm})A^*$. First, the momentum distribution (MD) of collective pions has a bright maximum at $K \simeq 0.3$ GeV/c. Second, the excitation spectrum of the recoil nucleus is concentrated at low energies $E_{\mathrm{rec},A} \simeq K^2/2M_A \leq$ 1 MeV. The results for the pion knockout from mesonic clouds of individual nucleons in nuclei are distinctly different.

PACS. 25.30.Rw Electroproduction reactions – 13.60.Le Meson production – 13.75.Gx Pion-baryon interactions

1 Introduction

On the basis of our previous papers [1,2] concerning an intermediate-energy reaction of quasielastic knockout of pions from the nucleon, we propose here a program of direct experimental investigation of pionic wave functions in nuclei by means of the quasielastic-knockout (QEK) reaction

$$A(N,Z)(e,e'\pi^-)A(N-1,Z+1,\omega)$$

or

$$A(N,Z)(e,e'\pi^+)A(N+1,Z-1,\omega)$$

initiated by electrons with the energy of a few GeV and mediated by longitudinal virtual photons with the squared mass Q^2 of 2–4 GeV/c. This reaction provides the opportunity to measure pionic momentum distributions corresponding to various values of final nucleus-spectator excitation energy ω. Such experiments are urgent now in connection with the long-standing discussion of the pionic condensate in nuclei [3,4]. Namely, we show that the MD of delocalized pions in modern collective models [3,4] based on the virtual excitation of the Δ-isobar are qualitatively different from the MD of pions localized on individual nucleons in the nucleus. We expect that the spectra of energies ω will also be different in these two cases —the quasielastic knockout of a pion from the pionic cloud of an individual nucleon in the nucleus and the quasielastic knockout of a pion from the nucleus's pionic condensate. The experimental energy resolution of 10 MeV would be quite efficient here.

The background of QEK in microphysics is very rich. The exclusive processes of the quasielastic knockout of protons from the atomic nucleus by protons $(p, 2p)$ or by electrons (e, ep) at bombarding energies of a few hundred MeV are well known [5]. They were used for investigations of the MDs of nucleons on different shell model orbitals. The shape of the MD for light nuclei proved to be very sensitive to values of the nucleon shell model quantum numbers nl.

In the QEK experiments, the coincidence technique is used with the energy resolution $\Delta E \sim 1$ MeV, and the interpretation of results is based on very simple binary conservation laws

$$E(\boldsymbol{p}) = E(\boldsymbol{p}') + E(\boldsymbol{k}') + E_{\mathrm{bind}}, \qquad \boldsymbol{p} + \boldsymbol{k} = \boldsymbol{p}' + \boldsymbol{k}', \quad (1)$$

which are valid here, because energies of both an initial bombarding particle and two final particles are high (the impulse approximation [6]). In eq. (1), $\boldsymbol{k}$ is the momentum of a virtual particle to be knocked out, E_{bind} its binding energy; $\boldsymbol{p}$ and $E(\boldsymbol{p})$ are the initial momentum and energy of the bombarding particle, $\boldsymbol{p}'$ and $E(\boldsymbol{p}')$ are the final momentum and energy of this particle; and $\boldsymbol{k}'$ and $E(\boldsymbol{k}')$ are the momentum and energy of the knocked-out particle. The kinematics of the above-mentioned process corresponds to the inequalities $|\boldsymbol{k}| \ll |\boldsymbol{p}|, |\boldsymbol{p}'|, |\boldsymbol{k}'|$ and $E_{\mathrm{bind}} \ll E(\boldsymbol{p}), E(\boldsymbol{p}'), E(\boldsymbol{k}')$. Thus, the values of $\boldsymbol{k}$ and E_{bind} are obtained from the experiment.

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One more nuclear example are the exclusive reactions of quasielastic knockout of nucleon clusters by protons with energy 0.5–1 GeV such as $(p, p\alpha)$, etc. This reaction may be important for the identification of de-excitation mechanisms of the virtually excited clusters in nuclei [7].

The QEK process $(e, 2e)$ at the beam energies of around 10 keV is widely used in investigations of the electronic structure of atoms, molecules and solids [8,9]. So, there exists a great experience in the investigation of the exclusive QEK reactions.

In our previous papers [1,2,10] we used the concept of the quasielastic knockout to the knockout of mesons from nucleons by high-energy electrons. It demanded a relativistic generalization of the theory. Namely, the t-channel pole z-diagram reflecting a virtual creation of, say, a $\pi^+\pi^-$ pair was taken into account in addition to the t-channel pole diagram of pion knockout (the instantaneous form of dynamics). The second important point mentioned there was that it was possible to separate experimentally reactions induced by longitudinal virtual photons γ_L^* and those induced by transverse virtual photons γ_T^* [11]. This offers a way [12] to investigate the MDs of pions ($\pi^{+*} + \gamma_L^* \to \pi^+$ subprocess) and the MDs of ρ-mesons ($\rho^{+*} + \gamma_T^* \to \pi^+$ subprocess) by means of the same $p(e, e'\pi^+)n$ reaction. The quasielastic kinematics requires the squared mass of a virtual photon Q^2 to be about 2–4 $(\mathrm{GeV}/c)^2$. Such Q^2 value is necessary to suppress the contribution of the competing s-channel pole diagram which corresponds to quite different physics and is very important when the real photon is absorbed.

In the present paper we extend this approach to the investigation of the pionic degrees of freedom in nuclei. The paper is organized as follows. In the second section we outline a relativistic formalism for the QEK reactions. In the third section we discuss the final-state interaction and its influence on the cross-section. In the fourth section, we present a simple expression for the MD of the collective pions in a model based on the virtual excitation of the Δ-isobar and compare it with the MD of pions localized on nucleons in a nucleus. In the fifth section we discuss the advantage of the $(e, e'\pi)$ process in comparison with the (γ, π) reaction and the $(\pi, 2\pi)$ QEK process.

2 Formalism

According to the general theory of meson electroproduction [13], the differential cross-section for the reaction $T + e \to R + \pi^+ + e'$ can be presented as a sum of four terms:

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}W^2\mathrm{d}Q^2\mathrm{d}t\mathrm{d}\phi_\pi} = \Gamma\left\{\varepsilon\frac{\mathrm{d}\sigma_L}{\mathrm{d}t} + \frac{\mathrm{d}\sigma_T}{\mathrm{d}t} + \sqrt{2\varepsilon(1+\varepsilon)}\frac{\mathrm{d}\sigma_{LT}}{\mathrm{d}t}\cos\phi_\pi + \varepsilon\frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t}\cos 2\phi_\pi\right\}, \quad (2)$$

where $\mathrm{d}\sigma_L/\mathrm{d}t$ corresponds to longitudinal virtual photons, $\mathrm{d}\sigma_T/\mathrm{d}t$ to transverse virtual photons, and $\mathrm{d}\sigma_{LT}/\mathrm{d}t$ and $\mathrm{d}\sigma_{TT}/\mathrm{d}t$ are interference components. The experimental cross-section can be separated into the longitudinal, transverse, and interference components by varying ε and ϕ_π (Rosenbluth separation) [11]. Experimental results are usually presented in terms of $\mathrm{d}\sigma_i/\mathrm{d}t$, $i = L, T, LT, TT$.

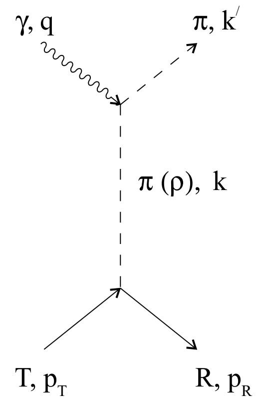

Fig. 1. t-pole diagram for the pion production: p_T, p_R are, respectively, 4-momenta of a target particle and of a recoil particle, k' is the 4-momentum of the final pion, q is the 4-momentum of a virtual photon, and k is the 4-momentum of a virtual meson.

In eq. (2), $W^2 = (q + p_T)^2 = (k' + p_R)^2$ is the square of the invariant mass, p_T, p_R are, respectively, 4-momenta of a target particle and of a recoil particle, k' is the 4-momentum of the final pion and q is the 4-momentum of the virtual photon $q = (q_0, \boldsymbol{q})$ (see fig. 1); $Q^2 = -q^2$; $t = (p_R - p_T)^2 = (k' - q)^2 = k^2$ ($k = (k_0, \boldsymbol{k})$ is the 4-momentum of a virtual meson); ϕ_π is the angle between the electron scattering plane and the plane spanned by the $(\boldsymbol{k}', \boldsymbol{p}_R)$ momenta; and

$$\Gamma = \frac{\alpha}{(4\pi)^2}\frac{W^2 - M_T^2}{Q^2E_e^2M_T^2}\frac{1}{1-\varepsilon}, \quad (3)$$

is the virtual-photon flux. Here E_e is the initial electron energy, M_T is a target mass, $M_T = M_N$ for the process on a free nucleon and $M_T = M_A$ for the process on a nucleus. The quantity

$$\varepsilon = \left[1 + \frac{2\boldsymbol{q}^2}{Q^2}\tan^2\frac{\theta_e}{2}\right]^{-1} \quad (4)$$

characterizes a degree of longitudinal polarization of the virtual photon (θ_e is the angle between the momenta of incident and scattered electrons).

As we pointed out in our previous papers [1,2], the longitudinal cross-section for quasielastic kinematics is dominated by the t-pole diagram fig. 1 with virtual pions (which corresponds to the subprocess $\gamma_L^* + \pi^* \to \pi$), and the transverse cross-section is dominated by the t-pole diagram fig. 1 with virtual ρ-mesons (the subprocess $\gamma_T^* + \rho^* \to \pi$). So, if we want to study the pionic cloud of the nucleon (nucleus), we need the experimental data on the longitudinal cross-section $\mathrm{d}\sigma_L/\mathrm{d}t$, and if we study the ρ-mesonic cloud we need the experimental data on the transverse cross-section $\mathrm{d}\sigma_T/\mathrm{d}t$.

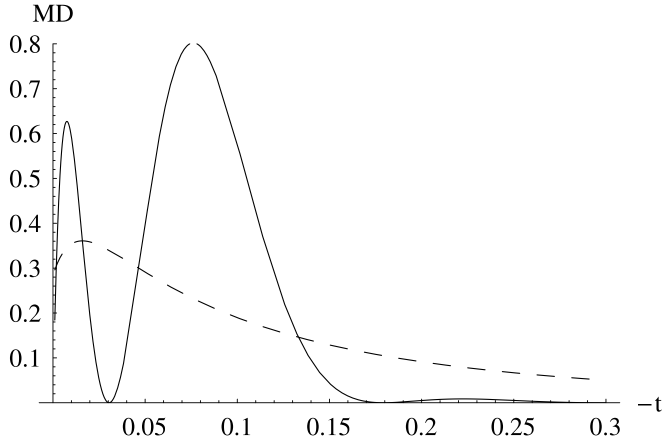

Fig. 2. Momentum distribution of pions *versus* $-t$, (GeV^2/c^2): the solid curve is the MD of the delocalized pions in nuclei (per nucleon) $\overline{|\Psi_A^{A\pi}(\boldsymbol{k})|^2}$, $(\mathrm{GeV}/c)^{-3}$; and the dashed curve is the washed-out MD of the localized pions in the nucleus $\overline{\overline{|\Psi_N^{N\pi}(\boldsymbol{k})|^2}}$, $(\mathrm{GeV}/c)^{-3}$.

Now, let us introduce the wave function of pions [1,2]

$$\Psi_T^{R\pi}(\boldsymbol{k}) = \frac{J(T \to R\pi)}{k_0 - E_\pi(\boldsymbol{k})}, \tag{5}$$

where $J(T \to R\pi)$ is the amplitude of the virtual transition $T \to R\pi$, $E_\pi(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2 + m_\pi^2}$, m_π is the pion mass.

For the kinematics of quasielastic knockout the longitudinal cross-section is directly proportional to the momentum distribution of pions (*i.e.*, the squared and spin-averaged wave function (5)) [1,2]:

$$\frac{\mathrm{d}\sigma_L}{\mathrm{d}t} = \frac{\alpha}{8}\frac{\overline{|\Psi_T^{R\pi}(\boldsymbol{k})|^2}}{(k_0 + E_\pi(\boldsymbol{k}))^2}\frac{1}{|\boldsymbol{q}^{(\mathrm{cm})}|}\frac{1}{W(W^2 - M_T^2)} \times F_\pi^2(Q^2)((k + k') \cdot e_{\lambda=0})^2, \tag{6}$$

where $\alpha = 1/137$; $\boldsymbol{q}^{(\mathrm{cm})}$ is the virtual-photon momentum in the center-of-mass frame,

$$|\boldsymbol{q}^{(\mathrm{cm})}| = \sqrt{Q^2 + (W^2 - Q^2 - M_T^2)^2/(4W^2)}; \tag{7}$$

$F_\pi(Q^2)$ is the pion electromagnetic form factor $F_\pi(Q^2) = [1 + (Q^2/0.5\ (\mathrm{GeV}/c)^2)]^{-1}$; $e_{\lambda=0}$ is the photon polarization unit 4-vector for longitudinal photons, and the scalar product is $((k + k') \cdot e_{\lambda=0})^2 = 4(k'_0 q_z - q_0 k'_z)^2/Q^2$.

The wave function (5) is normalized to the probability of finding a pion in the channel of the virtual decay $T \to R + \pi$ (spectroscopic factor):

$$\int \overline{|\Psi_T^{R\pi}(\boldsymbol{k})|^2}\mathrm{d}\tau = S_T^{R\pi} \tag{8}$$

with the integration measure

$$\mathrm{d}\tau = \mathrm{d}^3k/(4\pi)^3 M_T E_\pi(\boldsymbol{k}) E_R(\boldsymbol{k}). \tag{9}$$

We have included the factor

$$[(4\pi)^3 M_T E_\pi(\boldsymbol{k}) E_R(\boldsymbol{k})]^{1/2} \tag{10}$$

into the wave function, so that the wave functions presented in fig. 2 are normalized to the spectroscopic factor with the integration measure $\mathrm{d}\tau = \mathrm{d}^3k$.

3 Final-state interaction

Quasielastic-knockout experiments will not provide us with the pion momentum distribution $\overline{|\Psi_T^{R\pi}(\boldsymbol{k})|^2}$ itself, but with its distorted version [7,14] $\overline{|\phi_T^{R\pi}(\boldsymbol{k'q})|^2}$ due to the final-state interaction (FSI) of the knocked out pion with the residual nucleus. Namely, let us write down a plane-wave Fourier transformation of the wave function

$$\begin{aligned}\Psi_T^{R\pi}(\boldsymbol{k}) &= \frac{1}{(2\pi)^{3/2}}\int e^{-i\boldsymbol{kr}}\Psi_T^{R\pi}(\boldsymbol{r})\mathrm{d}^3\boldsymbol{r} \\ &= \frac{1}{(2\pi)^{3/2}}\int e^{i\boldsymbol{qr}}e^{-i\boldsymbol{k'r}}\Psi_T^{R\pi}(\boldsymbol{r})\mathrm{d}^3\boldsymbol{r},\end{aligned} \tag{11}$$

where $\boldsymbol{k} = \boldsymbol{k'} - \boldsymbol{q}$.

The electronic plane wave $e^{i\boldsymbol{qr}} = e^{i(\boldsymbol{p}_{e'} - \boldsymbol{p}_e)\boldsymbol{r}}$ as well as the wave function of a virtual pion $\Psi_T^{R\pi}(\boldsymbol{r})$ are not modified by the final pion-nucleus interaction, but the FSI remarkably affects the propagation of the final knocked-out pion in the nuclear medium. Instead of the plane-wave function $e^{-i\boldsymbol{k'r}}$ we should use a distorted function $\Psi^{(-)}(\boldsymbol{k'}, \boldsymbol{r})$. When $|\boldsymbol{k'}|$ is large enough (≥ 1 GeV/c), we can use the eikonal approximation [15] to obtain this function. Solving the Klein-Gordon equation in the eikonal approximation with a simplified optical pion-nucleus potential, which has the same depth $U = V + iW$ in the integration range of eq. (11), we obtain the distorted-wave function of the knocked-out pion:

$$\Psi^{(-)}(\boldsymbol{k'}, \boldsymbol{r}) = e^{(\boldsymbol{k'} - (V + iW)\hat{\boldsymbol{k}}')\boldsymbol{r}}. \tag{12}$$

So, in eq. (11), instead of the momentum $\boldsymbol{k'}$ we should use a "distorted local momentum" [7] $\boldsymbol{k'} - (V + iW)\hat{\boldsymbol{k}}'$, where $\hat{\boldsymbol{k}}'$ is the unit vector $\boldsymbol{k'}/|\boldsymbol{k'}|$,

$$\boldsymbol{k'} \to \boldsymbol{k'} - (V + iW)\hat{\boldsymbol{k}}'. \tag{13}$$

The usage of such potential, which is typical for infinite nuclear matter, is justified because the integration area in eq. (11) is cut off due to the localized character of the wave function of virtual pions in the nucleus $\Psi_T^{R\pi}(\boldsymbol{r})$. This simplified consideration permits us to see directly how the FSI affects the wave function of pions. Namely, with the distorted-wave function of the real pion, eq. (11) takes the form

$$\phi_T^{R\pi}(\boldsymbol{k'}, \boldsymbol{q}) = \frac{1}{(2\pi)^{3/2}}\int e^{i\boldsymbol{qr}}e^{-i(\boldsymbol{k'} - V\hat{\boldsymbol{k}}')\boldsymbol{r}}e^{-W\hat{\boldsymbol{k}}'\boldsymbol{r}}\Psi_T^{R\pi}(\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}. \tag{14}$$

Comparing eq. (11) and eq. (14), we can see that taking the FSI into account results in shifting of the wave function $\Psi_T^{R\pi}(\boldsymbol{k})$ to the left $\boldsymbol{k} \to \boldsymbol{k} + V\hat{\boldsymbol{k}}'$ and in its damping by a factor of the order e^{-WR}, where R is the nuclear radius.

In our calculations we used a square well with a finite radius R equal to the nuclear radius. In this case, the wave function of the knocked-out pion has the form

$$\Psi^{(-)}(\boldsymbol{k}',\boldsymbol{r}) = e^{i\boldsymbol{k}'\boldsymbol{r}}e^{iS(\boldsymbol{r})}, \tag{15}$$

with

$$S(\boldsymbol{r}) = -\frac{E_\pi(\boldsymbol{k}')}{|\boldsymbol{k}'|}\int_z^\infty U(\boldsymbol{b}+\hat{\boldsymbol{k}}'z')\mathrm{d}z'. \tag{16}$$

Here $\boldsymbol{b}$ is the pion impact parameter, and z is pion's coordinate along the direction of its propagation.

Pion-nucleus optical potentials considerably depend on the pion momentum [16,17]. But as far as the virtual-pions momenta lie in a very narrow range ($|\boldsymbol{k}|$ changes from 0 to 0.4 GeV/c —see fig. 2) and are very small in comparison with the final-pion momenta ($|\boldsymbol{k}'|$ is about 4–5 GeV/c (see below)), we may use an optical potential with the same depth for all final-pion momenta.

The parameters of the well depth at our energies ($|\boldsymbol{k}'| = 4$–5 GeV/c) are $V = 22$ MeV and $W = 53$ MeV [16,17]. A πA potential of this depth, having the Woods-Saxon shape in accordance with the nucleon distribution, describes, say, the π^--^{40}Ca scattering at 800 MeV and low scattering angles in the eikonal approximation [18].

4 Momentum distributions of pions in nuclei

The radial wave function of the collective pion appears as the standing P-wave [4]:

$$\begin{aligned} \Phi(r) &= cj_1(Kr), \qquad r \le R, \\ \Phi(r) &= 0, \qquad\qquad\; r > R. \end{aligned} \tag{17}$$

Here the collective pionic mode is characterized by the momenta of pions $\boldsymbol{K}$,

$$\sqrt{\boldsymbol{K}^2 + m_\pi^2} = M_\Delta - M_N, \qquad K \simeq 0.3 \ \ \mathrm{GeV}/c, \tag{18}$$

M_Δ and M_N being the masses of the Δ-isobar and the nucleon.

It is supposed that initial and final nuclear states have the same parity and transitions like $0^+ \to 1^+$, $1^+ \to 1^+$, etc. take place. The situation is close to the weak coupling of pions to the nucleus. In the above equation, R denotes the nucleus radius. The nucleus should have $A \simeq 70$ and $N \simeq Z$ to obey the qualitative estimations of the number of collective pions in the nucleus per nucleon $n_{i,\mathrm{coll}} \approx 0.1$ [4]. This estimation corresponds to the value $c = 0.027$ of the normalization constant, and the quasielastic-knockout experiment will verify this value. By the Fourier transformation of the single-particle wave function (17), we obtain the MD of the collective pions $\overline{|\Psi_A^{A\pi}(\boldsymbol{k})|^2}$ (the squared and spin-averaged wave function in the momentum representation). This MD is presented in fig. 2 (solid curve). It has a sharp maximum at $k = K$, *i.e.* it is close to the plane wave within the limitation imposed by the finite volume of the nucleus.

The dashed line corresponds to a washed-out MD of the localized pions $\overline{\overline{|\Psi_N^{N\pi}(\boldsymbol{k})|^2}}$. The MD of the pion in a free nucleon $\overline{|\Psi_N^{N\pi}(\boldsymbol{k})|^2}$ was reconstructed from the $p(e,e'\pi)n$ experiment [11] with the quasielastic kinematics ($Q^2 = 1$–3 (GeV/c)^2) and was presented in our previous papers [1,2]. In order to obtain the MD of the localized pions we should average this MD over the motion of a nucleon in the nucleus. So, the washed-out MD of the localized pions was obtained by the convolution of the MD of pions in a free nucleon with the averaged MD of shell model nucleons in the nucleus $\overline{|\Phi_A^{A-1,N}(\boldsymbol{p})|^2}$:

$$\overline{\overline{|\Psi_N^{N\pi}(\boldsymbol{k})|^2}} = \int \overline{|\Psi_N^{N\pi}(\boldsymbol{k}+(m_\pi/M_N)\boldsymbol{p})|^2}\cdot\overline{|\Phi_A^{A-1,N}(\boldsymbol{p})|^2}\mathrm{d}\boldsymbol{p}. \tag{19}$$

The washed-out MD of the localized pions, in contrast to the MD of the collective pions is very smooth at k values close to K. The momentum distributions in fig. 2 are isotropic with respect to $\boldsymbol{k}$ direction, because they are averaged over magnetic quantum numbers of the pionic P-orbital.

The knockout of the delocalized collective pions is accompanied by recoil to the final nucleus as a whole, with a very small recoil energy $E_{\mathrm{rec},A} \simeq K^2/2M_A < 1$ MeV ($A \simeq 70$–80), although the momentum K itself is not small (see eq. (4)). The wave function of pions (17) does not contain pion-nucleon spatial correlations and, as a result, the final recoil nucleus will not be internally excited. In fact, the best experimental energy resolution ΔE at present may be around 10 MeV here. The real situation will correspond to the summation over many excited states of external shells of the final nucleus, *i.e.* to a sum rule.

At the same time, knockout of pions with the same virtual momentum $\boldsymbol{k}$, $k \simeq K$ from the pion cloud of an individual nucleon will be characterized by a large value of the recoil energy transferred to one nucleon, $E_{\mathrm{rec},N} \simeq K^2/2M_N \simeq 50$ MeV. This nucleon, with a high probability, will be directly emitted from the nucleus (we mean here the numerous weakly bound nucleons of the external shell). A reliable identification of this event requires triple coincidences $e' + \pi^- + p$, which is an urgent experimental problem now. In a noncomplete experiment with only double coincidences $e' + \pi^-$, the above-mentioned event will be perceived as a high excitation ($\omega \simeq 50$ MeV) of the final recoil nucleus accompanied by transfer of the momentum $-\boldsymbol{k}$, $k \simeq K$ to this nucleus. This group of events will show a very smooth MD of virtual pions like the corresponding part of the dashed line in fig. 2 at k around K. The discussed $n \to p$ transformation with the recoil to the proton and the corresponding appearance of a particle-hole pair in the process of the π^- knockout from the nucleus may also create one of the charged nuclear giant resonances with the excitation energy ω of a few tens of MeV. An intriguing new opportunity here is to investigate the k-dependence of such cross-sections.

Figure 3 represents the influence of the final-state interaction on the cross-section in the case of delocalized pions. As we have mentioned above, the final-state interaction causes shifting and damping of the cross-section.

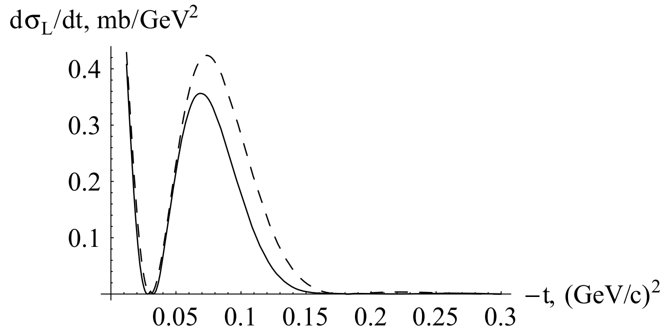

Fig. 3. Influence of the final-state interaction on the cross-section. Longitudinal cross-section $d\sigma_L/dt$, (mb/GeV^2), *versus* $|t|$, (GeV/c)^2 for delocalized pions. $Q^2 = 1$ (GeV/c)^2, $q_0^{(\text{lab.s.})} = 4.86$ GeV. The solid line corresponds to the calculation involving the final-state interaction, and the dashed line, to the plane-wave approximation.

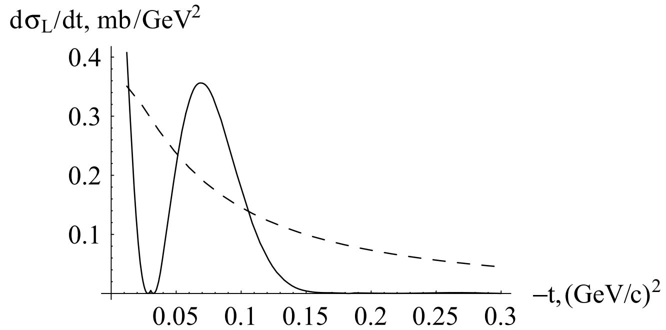

Fig. 4. Longitudinal cross-section $d\sigma_L/dt$, (mb/GeV^2), *versus* $|t|$, (GeV/c)^2. $Q^2 = 1$ (GeV/c)^2, $q_0^{(\text{lab.s.})} = 4.86$ GeV. The dashed line corresponds to localized pions, and the solid line, to delocalized pions. The final-state interaction is taken into account.

However, taking the interaction into account does not distort the cross-sections dramatically, so we are still able to discriminate between the two kinds of pions in the nucleus.

Theoretically predicted cross-sections for the both cases (localized and delocalized pions), with the final-state interaction taken into account, are shown in fig. 4. Note that the cross-section eq. (6) is calculated with the wave function of pions (5), which obey eq. (8). So, calculating the cross-section with the collective-pion wave function (17), this radial wave function should be divided by $\sqrt{4\pi}$ and by the factor (10).

The above discussion should be complemented by a remark about the minimal value of t available in the experiment with the given values of Q^2 and q_0. Namely, from the expression $t = -k^2 = -(k' - q)^2$ we obtain for the cosine of the angle between $\boldsymbol{k}'$ and $\boldsymbol{q}$

$$\cos\theta = \frac{-t - m_\pi^2 + Q^2 + 2q_0(q_0 - t/2M_T)}{2|\boldsymbol{q}|\sqrt{(q_0 - t/2M_T)^2 - m_\pi^2}} \simeq \frac{-t + Q^2 + 2q_0^2}{2|\boldsymbol{q}|q_0}. \tag{20}$$

The condition $|\cos\theta| \le 1$ determines the range of physical t values. This gives the minimal t

$$t_{\min} \simeq (|\boldsymbol{q}| - q_0)^2. \tag{21}$$

In the case of virtual pions this formula gives the following expression for the minimal momentum of the virtual pion:

$$|\boldsymbol{k}|_{\min} \simeq \frac{Q^2}{2|\boldsymbol{k}'|}. \tag{22}$$

Figure 2 shows that all characteristic features of the MD of delocalized pions lie in the region of small t ($t < 0.1$ (GeV/c)^2). The smaller $t_{\min}$ we want to achieve, the smaller the difference $|\boldsymbol{q}| - q_0$ should be. At the same time, Q^2 and q_0 should be large enough to provide the kinematics of quasielastic knockout. For example, if $t_{\min} \simeq 0.01$ (GeV/c)^2, we can choose $Q^2 = 1$ (GeV/c)^2 and $q_0 \simeq 5$ GeV/c.

The principal result of this section is that the high-energy pion electroproduction on nuclei by means of the virtual longitudinal photons γ_L^* within the kinematics of the QEK process at small ω values of a few MeV offers an opportunity to see the cooperative, maximally delocalized pions in nuclei in the most direct way. The bright MD maximum at $k \simeq K \simeq 0.3$ GeV/c (the solid line in fig. 4) will be the principal indication of the presence of such pions in the nucleus. The increase of ω corresponds, qualitatively, to the increase of localization of the discussed virtual pions in the nucleus. The shape of the MD measured at different ω may be helpful for the clarification of the evolution of the reaction mechanism (the usage of the triple coincidence would be very important here, too).

It should be noted, that the final energy of a knocked-out pion should not be lower than 1 GeV to avoid disturbing influence of the intermediate resonance (Δ-isobar) in the πA final-state interaction. Energies significantly higher than 1 GeV are not suitable either, because they represent different physics of asymptotical quark counting rules [19]. This physics, which corresponds to the Q^2 values of 10–20 (GeV/c)^2 and rather small cross-sections, is very popular now [20]. But our region of the soft hadronic degrees of freedom in the nucleons and nuclei, which corresponds to rather moderate Q^2 values of 1–4 (GeV/c)^2 and which is still in the shadow, is not less interesting. In addition, it corresponds to quite measurable cross-sections.

5 Conclusion

In this paper we have shown that the reaction of pion quasielastic knockout from nucleus by high-energy electrons is an efficient tool to discriminate between the MD of delocalized pions corresponding to collective models of pion condensate, and the MD of pions localized on nucleons in the nucleus. We have shown that the final-state interaction between the knocked-out pion and the residual nucleus-spectator does not reduce significantly the sensitivity of the differential cross-section to the shape of the pionic MD. It is expected that the spectra of excitation energies of the final nucleus-spectator, ω, will also be different in these two cases.

Recently, the investigation of the $(\pi, 2\pi)$ quasi-elastic-knockout process has been initiated [21], although the energies of the pions are not yet high enough. The cross-sections here are larger (strong interaction) than those for the $(e, e'\pi)$ reaction, and the $(\pi, 2\pi)$ reaction gives an opportunity to investigate the π^0-component of the collective field. However distortion and absorption effects [22] for three pionic waves in the $(\pi, 2\pi)$ reaction are much more pronounced than those for one pionic wave in the $(e, e'\pi)$ process. These two reactions, the $(e, e'\pi)$ reaction of volume character with smaller cross-sections and the $(\pi, 2\pi)$ reaction of surface character with larger cross-sections, may complement each other rather efficiently.

Finally, it should be noted, that the process of pion photoproduction (γ, π) on nuclei ($Q^2 = 0$) is described by the interference of a number of amplitudes corresponding to different diagrams [23] and does not offer a direct way for extracting the MDs of pions in nuclei.

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